In the Name of God

Sharif University of Technology

Graduate School of Management and Economics

Macroeconomics 2 - 2023

Problem Set 10

1 A Mini Survey of The Main Results from Fiscal Policy

Are the following statements true or false? Briefly explain your reasoning, based on the models we discussed in class.

1. Suppose the government finances its spending solely by lump-sum taxes. Since this will take away some resources from the representative consumer, it will decrease both her consumption and investment at steady state.

2. Suppose the government finances all its spending by income taxes. That is, the consumers need to pay a fraction τ of their income to the government. As τ is increased, government revenue at steady state also increases.

3. Suppose the government finances all its spending by income taxes. Since the government taxes are levied only on income, the steady-state consumption level is not affected as the government increases the tax rate.

4. Suppose the government finances its spending by income taxes and debt. If the government needs to finance a temporary increase in government spending because of a hurricane, this should optimally be financed only by a temporary increase in the tax rate so that the level of government debt continues to stay at the same level.

5. Suppose the Ricardian equivalence holds in an economy. If the government wants to increase private consumption this year, then it is a desirable fiscal policy to tax less this year by borrowing more (i.e., by issuing government bonds), so that the consumer has more resources to use for consumption.

2 Determining The Catastrophe Date, T (1)

This question is based on the monetary-fiscal model we talked about in class. In this problem, assume that the GDP per capita also grows at rate g.

1. Show that the debt-to-GDP ratio, b_t, in an economy is given by the following equation:

$$b_t = \left(d - \frac{\mu}{1+\mu}\right) \sum_{i=0}^{t} \gamma^i$$

   with $\gamma = \frac{1+r}{1+n} > 1$.

2. Given that $\sum_{i=0}^{\infty} \alpha^i = \frac{1}{1-\alpha}$ for $0 < \alpha < 1$, prove that $\sum_{i=0}^{T} \gamma^i = \frac{1-\gamma^{T+1}}{1-\gamma}$

3. Use the above finite geometric series formula that you proved to calculate the catastrophe date, T, as a function of the model's parameters.

4. How does T change as $\bar{b}$ increases? Briefly explain the intuition for the observed change.

5. How does T change as μ increases? Briefly explain the intuition for the observed change.

6. How does T change as d increases? Briefly explain the intuition for the observed change.

7. Solve for the money growth rate $\bar{\mu}_t$ for $t > T$. and show that $\mu < \mu^* < \bar{\mu}_t$.

8. (Optional) Calibrate your model with some reasonable numbers (It means that use some sensible numbers for the parameters of the model). Find T numerically. Find $\bar{\mu}$ and μ^*.

9. (Optional) Run a sensitivity analysis on the results you found the previous part. It means that keep all the parameters fixed, then vary one parameter and look at the effects on the variables of interest which are $T, \bar{\mu}$ and μ^*.

10. The current analysis does not have any rational expectations in place. Explain intuitively how would our results change if we have rational expectations too.

11. (Optional) In this setup, we had a very simple money demand function. Explain intuitively how would the results change if the money demand function is negatively related to the nominal interest rates (or inflation).

3 Determining The Catastrophe Date, T (2) (Optional)

Consider the monetary-fiscal model we talked about in class, except assume that $\frac{M_t}{P_t} = Y_t (1+R)^{-\eta}$. Assume that the GDP per capita, and populatins grows at rate g and n.

1. Solve for the debt-to-GDP ratio, b_t, in this economy

2. Find the catastrophe date, T, as a function of the model's parameters (at least numerically)

3. Solve for the money growth rate $\bar{\mu}_t$ for $t > T$.

4 Optimal Monetary Policy 1

In this problem, we want to analyze how discretionary policy and rule-based policy differ. Suppose the Philips curve is

$$y_t = y_n + b\left(\pi_t - E_t\left[\pi_{t+1}\right]\right)$$

where y_t is the output gap and π_t is the inflation rate. The central bank sets the inflation rate such that

$$L_t = \frac{1}{2}\left(a\left(\pi_t - \pi^T\right)^2 + \left(y_t - y_n - k\right)^2\right)$$

is minimized where k is a desirable level of output above the natural rate and π^T is an inflation target.

1. Derive the optimal discretionary policy. How does optimum inflatin level depend on k, π^T, a, b?

2. Derive the optimal policy rule for the rule-based central banker. How is your solution different from the discretionary policy? How does optimum inflation level depend on k, π^T, a, b?

3. (Optional - Extra Credit) How does your results change if $L_t = \frac{1}{2}E_t\left[\sum_{i=0}^{\infty}\beta^i\left(a\pi_{t+i}^2 + y_{t+i}^2\right)\right]$

5 (Optional) Optimal Monetary Policy 2

(Hint: Refer to the Walsh Book, chapter 7)

Suppose that the economy is described by the following log-linearized equations

$$\begin{aligned} x_t &= E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) + \left(E_t z_{t+1} - z_t\right) + u_t \\ \pi_t &= \beta E_t\left[\pi_{t+1}\right] + \kappa x_t + e_t \end{aligned}$$

where x_t is the output gap and π_t is the inflation rate. Also u_t is a demand shock, z_t is a productivity shock, and e_t is a cost shock. Assume that

$$\begin{aligned} u_t &= \rho_u u_{t-1} + \xi_t \\ z_t &= \rho_z z_{t-1} + \phi_t \\ e_t &= \rho_e e_{t-1} + \varepsilon_t \end{aligned}$$

where $\xi_t, \phi_t, \varepsilon_t$ are white noise processes. The central bank sets the nominal interest rate i_t to minimize

$$L_t = \frac{1}{2} E_t \left[\sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2 \right) \right]$$

1. Interpret each equation.
2. Derive the optimal time-consistent policy for the discretionary central banker. Write down the FOCs and the reduced-form solutions for x_t and π_t.
3. Derive the interest rate feedback rule implied by the optimal discretionary policy.
4. Show that under the optimal policy, nominal interest rates are increased enough to raise the real interest rate in response to a rise in expected inflation. How will x_t and π_t move in response to a demand shock? A productivity shock? Briefly explain.
5. How would money supply look like?
6. Now suppose $z_t = u_t = 0$. Derive the optimal policy rule for the rule-based central banker. How is your solution different from the discretionary central banker?
7. (Optional) How does your results change if $L_t = \frac{1}{2}\left[\left(\pi_t^2 + \lambda x_t^2\right)\right]$

6 Solving the dynamic-inconsistency problem through punishment

Consider an infinite period version of the model by Barro and Gordon (1983). A policymaker has an objective function of the form

$$W = \sum_{i=0}^{\infty} \beta^i \left(y_i - \frac{1}{2} a \pi_i^2 \right)$$

to be optimized, where $a > 0$ and $0 < \beta < 1$. Assume $y_t = \bar{y} + b(\pi_t - \pi_t^e)$ which is the Lucas Supply function. Expected inflation is determined as follows: If π has equaled $\hat{\pi}$ (where $\hat{\pi}$ is a parameter) in all previous periods, then $\pi^e = \hat{\pi}$: If ever differs from $\hat{\pi}$; then $\pi^e = \frac{b}{a}$ in all subsequent periods.

1. What is the equilibrium of the model in all subsequent periods if π ever differs from $\hat{\pi}$?
2. Suppose π has always been equal to $\hat{\pi}$; so $\pi^e = \hat{\pi}$: If the monetary authorities chooses to depart from $\pi = \hat{\pi}$; what value of π does it choose? What level of its lifetime objective function does it attain under this strategy? If the monetary authority continues to choose $\pi = \hat{\pi}$ every period, what level of its lifetime objective function does it attain?
3. For what values of $\hat{\pi}$ does the monetary authority choose $\pi = \hat{\pi}$? Are the values of a, b and β such that if $\hat{\pi} = 0$; the monetary authority chooses $\pi = 0$?
4. Disuss the effects of a, b and β on your results.