Government Spending in a Simple Model of Endogenous Growth

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Introduction

- Recent models of economic growth can generate long-term growth without relying on exogenous changes in technology or population.
- Some of the models amount to theories of technological progress (Romer 1986).
- ... and others to theories of population change. (Becker and Barro 1988).
Introduction

• In models that Private and social returns to investments diverge, decentralized choices lead to suboptimal rates of saving and economic growth (Arrow 1962; Romer 1986).

• private returns to scale may be diminishing, but social returns can be constant and increasing. (reflect spillovers of knowledge or other externalities )

• The present analysis incorporate a public sector into a simple, constant-returns model of economic growth.
Introduction

- Constant Returns to a broad concept of capital
- Include tax-financed government services that affect production and utility.

Growth and saving rate:
- Utility-type expenditures VS. productive government expenditures
- Income Tax (distortions) VS. consumption Tax (Lump sum)
- Optimizing household VS. Planning Government (consider Externalities)
Decentralized Economy

• Each representative household maximizes his overall utility

(1) \[ U = \int_0^\infty u(c)e^{-\rho t} dt \] in which:

(2) \[ u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0 \] iso-elastic utility function

• Each firm maximizes profit
• Population growth is zero (n = 0).
Decentralized Economy

• Each person works a given amount of time; that is, there is no labor-leisure choice.

• The maximization of the representative household’s overall utility in Equation (1) implies:

\[
\frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot (f' - \rho)
\]
Constant return to a broad concept of capital

- We assume (5) \( y = Ak \) thus \( f' = A \).

- Human and non-human capital
  - Human investments like: education and training
  - Diminishing with respect to each type of capital but constant return with two types together

- We can extend model to distinguish between two types of capital like Lucas (1988) for sectors to produce each type of capital
AK model

- Household’s maximization leads to Euler Equation:

\[
\frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot (f' - \rho)
\]

- We assume; (5) \( y = Ak \), \( A > 0 \)
- \( A \) is the constant net marginal product of capital; \( f' = A \).
- So we have \( \gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot (A - \rho) \)

\( \gamma \) denotes per capita growth rate.
AK model

• Assume that the technology is sufficiently productive to ensure positive steady-state growth, but not so productive as to yield unbounded utility;

\[ A > \rho > A(1 - \sigma) \]  

(7)

• In this model the economy is always at a position of steady-state in which all variables - c, k and y - grow at the rate \( \gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot (A - \rho) \)
Why is $k$ always in steady state? (proof by the group)

- We have
  \[
  \dot{k} = y - c - (x + n + \delta)k = Ak - c
  \]
- So we have a differential equation in $k$ that leads to the solution:
  \[
  k(t) = k(0)e^{\gamma t}
  \]
- Where
  \[
  c(t) = c(0)e^{\gamma t}
  \]
\[ \dot{k} = Ak - c \]

- So we have:

\[ (A - \gamma)k(0)e^{\gamma t} = c(0)e^{\gamma t} \]

- And so:

\[ (8) \quad c(0) = k(0) \cdot (A - \gamma) \]
Including Public Sector

- Let $g$ be the quantity of public services provided to each household-producer.
- We consider initially the role of public services as an input to private production.
- Production now exhibits constant returns to scale in $k$ and $g$ together, but diminishing returns in $k$ separately;

*Even with a broad concept of private capital, production involves decreasing returns to private inputs if the (complementary) government inputs do not expand in parallel manner.*
In a empirical study, Aschauer (1988) argues that the services from government infrastructure are particularly important in this context.

Why Is Infrastructure Important?

David Alan Aschauer*
Including Public Sector

• Given constant returns to scale the production function can be written as:

\[(9) \quad y = \Phi(k, g) = k \cdot \phi \left(\frac{g}{k}\right), \phi' > 0 \text{ & } \phi'' < 0\]

• Assumptions about production function

\[(10) \quad \frac{y}{k} = \phi \left(\frac{g}{k}\right) = A \cdot \left(\frac{g}{k}\right)^\alpha, \quad 0 < \alpha < 1\]
1) Flow of services need not correspond to government purchases.

But conceptually, it is satisfactory to think of the government as doing no production and owing no capital.

Then the government just buys a flow of output (Including services of highways, sewers, battleship, etc.) from the privat sector.
2) The total of government purchases, rather than the amount per capita, matters for each individual if public services are *nonrival* for users (as is true, e.g., Space programs). At least since Samuelson (1954), this element is important for determining the desirable scale of government activity. My view is that few actual government services (including, as Thompson [1974] argues, national defense) are *nonrival*. 
In economics, a **public good** is a good that is both non-excludable and non-rivalrous in that individuals cannot be effectively excluded from use and where use by one individual does not reduce availability to others. Examples of public goods include fresh air, knowledge, lighthouse, national defense, flood control systems and street lighting. Public goods that are available everywhere are sometimes referred to as global public goods.
The general idea of including $g$ as a separate argument of the production function is that private inputs, represented by $k$, are not a close substitute for public inputs.

Private activity would not readily replace public activity if user charges were difficult to implement, as in the case of such nonexcludable services as national defenses and the maintenance of law and order.

In other cases, users charges would be undesirable, either because the services or because external effects cause private production to be too low. (basic education)
Flat rate Income tax

(11) \[ g = T = \tau y = \tau \cdot k \cdot \phi \left( \frac{g}{k} \right) \]

- \( \tau \) be constant
- Balanced budget
- Each firm views \( g \) as given (constant)
  - Two views on \( g \) be constant

- A firm sets \[ \frac{\partial y}{\partial k} = r + \delta = r. \]
(12) \[ \frac{\partial y}{\partial k} = \phi\left(\frac{g}{k}\right) \cdot \left(1 - \phi' \cdot \frac{g}{y}\right) \]
\[= \phi\left(\frac{g}{k}\right) \cdot (1 - \eta) \quad , 0 < \eta < 1 \]

- And \( \eta \) is a constant equal to \( \phi' \cdot \frac{g}{y} \)
- Which is equal to \( \alpha \) and is actually elasticity of \( y \) with respect to \( g \) - Prove on board
Euler Equation

• With the presence of income tax we have

\[
\frac{\dot{c}}{c} = \frac{(1-\tau)r-\rho}{\sigma} = \frac{(1-\tau)\frac{\partial y}{\partial k}-\rho}{\sigma}
\]

which is equal to

\[
(13) \quad \gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot \left[(1 - \tau) \cdot \phi \left(\frac{g}{k}\right) \cdot (1 - \eta) - \rho\right]
\]

• Comparison with AK model

\[
\frac{g}{k} \text{ is constant because } \frac{g}{k} = \tau \text{ and there is a unique solution to this (reason)}
\]
Why is $k$ always in steady state? (proof by the group)

- We have $\dot{k} = y - g - c$
- So we have a differential equation in $k$ that leads to the solution:
  $$k(t) = k(0)e^{\gamma t}$$
- Where $c(t) = c(0)e^{\gamma t}$
- Again we have:

$$(14) \quad c(0) = k(0) \cdot \left[ (1 - \tau) \cdot \phi \left( \frac{g}{k} \right) - \gamma \right]$$
Optimum Size Of Government

\[
\gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot \left[ (1 - \tau) \cdot \phi \left( \frac{g}{k} \right) \cdot (1 - \eta) - \rho \right]
\]

• Two different effect of tax rate
• We have

\[
\frac{d\gamma}{d(g/y)} = \frac{1}{\sigma} \cdot \phi \left( \frac{g}{k} \right) \cdot (\phi' - 1)
\]

• The size of government that maximizes the growth:
  \[\phi' = 1\]
• Since \(\alpha = \eta = \phi' \cdot \left( \frac{g}{y} \right)\) then \(\alpha = \left( \frac{g}{y} \right) = \tau\)
• The growth rate is positive over some range if the economy is sufficiently productive relative to the rate of time preference.

• The condition for a range of positive growth is

\[ A^{1-\alpha} \cdot (1 - \alpha)^2 \cdot \alpha^{\frac{\alpha}{1-\alpha}} > \rho \]
I assume that the economy is not so productive that it allows the attained utility to become unbounded; the condition that can prove readily here is:

\[ \rho > A^{1-\alpha} \cdot (1 - \sigma)(1 - \alpha)^2 \cdot \alpha^{(1-\alpha)} \]
Saving rate \((S)\)

\[
S = \frac{\dot{k}}{y} = \frac{k}{k} \cdot \frac{\dot{k}}{y} = \frac{\gamma}{\phi\left(\frac{g}{k}\right)}
\]

Because \(k/y\) which is equal to reciprocal of \(\phi\left(\frac{g}{k}\right)\) declines with \(g/y\), the saving rate peaks before the optimum growth rate.
Why Maximizing growth?

• There is no reason for the government to maximize \( y \) or \( s \) per se. For a benevolent government, the appropriate objective in this model is to maximize the utility attained by the representative household.

\[
U = \int_0^\infty u(c)e^{-\rho t}dt,
\]

\[
u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}
\]

• We have (17) \( U = \frac{[c(0)^{1-\sigma}]}{(1-\sigma[\rho-\gamma(1-\sigma)])} \)

• (18) \( c(0) = \frac{k(0)}{1-\eta} \cdot [\rho + \gamma \cdot (\sigma + \alpha - 1)] \)

• (19) \( U = \left[\frac{k(0)}{1-\eta}\right]^{1-\sigma} \cdot \left\{\frac{\rho+\gamma(\eta+\sigma-1)^{1-\sigma}}{(1-\sigma)[\rho-\gamma(1-\sigma)]}\right\}^1 \)
Is Solution Pareto Optimal?

- The results on the size of government in the previous section are solutions to second-best policy problems. Because of familiar externalities implied by public expenditures and taxation, the decentralized choices of saving turn out to generate outcomes that are not Pareto optimal.

- The easiest way to assess the external effects is to compare the decentralized outcomes with those from an unrealistic planning problem.

- Suppose that the government chooses a constant expenditure ratio, $g/y$, and can then dictate each household's choices for consumption over time.
Solution for Planning problem (proof by the group)

- Maximize

- (1) \( U = \int_0^\infty u(c)e^{-\rho t}dt \), (2) \( u(c) = \frac{c^{1-\sigma-1}}{1-\sigma} \)

- W.r.t \( \dot{k} = A\left(\frac{g}{k}\right)^\alpha k - c - g \)

- \( H = u(c) + v(A\left(\frac{g}{k}\right)^\alpha k - c - g) \)

- So \( u'(c) = v \) and \( A\left(\frac{g}{k}\right)^\alpha + A\alpha\left(\frac{g}{k}\right)^{\alpha-1}k = -\dot{v} + \rho v \) and \( A\left(\frac{g}{k}\right)^{\alpha-1} = 1 \) so \( \left(\frac{g}{k}\right) \) is constant.

- By substituting two first equations we have...
The term inside the brackets and to the left of the minus sign is the \textit{social marginal return on capital}, given that the expenditure ratio, $g/y$, is constant.

The condition $g/k = (g/y) \cdot \varphi(g/k)$ implies that the derivative of $\gamma_p$ from equation (20) with respect to $g/y$ is:

$\frac{d \gamma_p}{d(g/y)} = \frac{\varphi \left( \frac{g}{k} \right) \cdot (\phi' - 1)}{\sigma(1 - \eta)}$

The condition $\varphi' = 1$ corresponds to maximum growth irrespective of the form of the production function.
Centralized plan
• replacing the income tax with a lump-sum tax in an environment of decentralized households:

\[
\gamma_L = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot \left[ \phi \left( \frac{g}{k} \right) \cdot (1 - \eta) - \rho \right]
\]

• With lump-sum taxes, the private marginal return on capital is \(\delta y/ \delta k\) rather than \((1 - \iota) \cdot (\delta y/ \delta k)\)
Lump-sum tax
As is apparent from equation (22), \( \gamma_L \) is monotonically increasing in \( g/y \) because a higher \( g/y \) means a higher marginal product of capital, \( \delta y/\delta k \).

With a lump-sum tax, households respond to the higher return on capital by choosing a higher growth rate for consumption.

A comparison of equations (20) and (22) indicates that \( \gamma_p \) contains the term \( 1 - (g/y) \) where \( \gamma_L \) contains the term \( 1 - \eta \).

\[
\begin{align*}
(20) \quad \gamma_p &= \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot \left[ \left( 1 - \frac{g}{y} \right) \cdot \phi \left( \frac{g}{k} \right) - \rho \right] \\
(22) \quad \gamma_L &= \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot \left[ \phi \left( \frac{g}{k} \right) \cdot (1 - \eta) - \rho \right]
\end{align*}
\]

\( \eta = \phi' \cdot \left( \frac{g}{y} \right) \) then \( \phi' = 1 \) then \( \eta = \left( \frac{g}{y} \right) \) Therefore, the terms \( 1 - (g/y) \) and \( 1 - \eta \) coincide at this point.
Another source of Distortion

- If the expenditure share is set non-optimally so that $\phi' \neq 1$, the planning solution for consumption does not coincide with the solution under lump-sum taxation.

- This result indicate that the income tax is not the only distortion in the model.

- An individual Producer computes the marginal product while holding constant the quantity of public services that he receives from the government.
If the government sets a given expenditure ratio \(\frac{g}{y}\), an increase in national product by one unit induces the government to raise the aggregate of its public services by \(\frac{g}{y}\) units.

Thus when an individual producer decides to raise his individual \(k\) and \(y\), he is indirectly causing the government to increase its aggregate spending.

The effect on that individual’s public services, which entered into his production function, would be negligible and therefore be ignored.

But it is nevertheless true, with \(\frac{g}{y}\) fixed, that an individual’s decision that raises national product by one unit causes the total of government purchases to expand by \(\frac{g}{y}\) units.

The effects depend on whether the size of the government is optimal.
Tax systems and property rights

- From the endpoint of investors, enhanced property rights look like reductions in marginal tax rates. Therefore, an improvement in property rights cause the rates of growth and saving increase.
An alternative Specification for public services

• Thus far, each individual held fixed his quantity, \( g \), when considering a change in his quantity of capital, \( k \), and output, \( y \). This setting is appropriate for some public services but not for others.

• For example, for police and fire protection, and perhaps for national defense, the amount of public services that an individual receives is roughly proportional to the amount of property that the person has to protect.
These cases can be approximated by assuming that each individual holds constant his ratio of public services to output, \( g/y \), rather than his level of public service.

With a flat-rate income tax at rate \( \tau \), the individual’s optimization problem now coincides with the planner’s problem considered before.
Government Consumption Services

Suppose that the government's expenditures also finance some services that enter into households' utility functions.

\[ u(c, h) = \frac{(c^{1-\beta} \cdot h^\beta)^{1-\sigma}}{1-\sigma} - 1 \]

We assume that total spending per household is \( g + h \), where the quantity \( h \) represents the government's consumption services. Assume a flat-rate income tax, so that the government's budget constraint is:

\[ T = (\tau_g + \tau_h) \cdot y \]

\[ \tau_g = \frac{g}{y}, \quad \tau_h = \frac{h}{y} \]
Households' decentralized choices

\[
\gamma_h = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot \left[ (1 - \tau_g - \tau_h) \phi \left( \frac{g}{k} \right) \cdot (1 - \eta) - \rho \right]
\]

For a given \( \iota_h \) and a Cobb-Douglas production function, it is easy to show that the value of \( \iota_g = g/y \) that maximizes \( \gamma_h \) in equation (25) is \( \alpha(1 - \iota_h) \)

- Proof By the group:
  - Let \( \frac{g}{k} = J(\iota_g) \)
  - First from the equation \( J = \iota_g \phi(J) \) it can be proven that \( J' = \frac{\phi}{1 - \iota_g \phi'} \)
  - First order condition:
    \[
    \phi = (1 - \iota_g - \iota_h) \phi' J' = (1 - \iota_g - \iota_h) \phi' \frac{\phi}{1 - \iota_g \phi'} 
    \]
    so \( \phi'(1 - \iota_g - \iota_h) = 1 - \iota_g \phi' \) and the result follows from \( \iota_g \phi' = \alpha \)
Why maximize growth?

• However, this choice turns out not to maximize the utility attained by the representative household.

• With these results, it is feasible to relate the attained utility, $U$, to the expenditure ratios $\frac{g}{y}$ and $\frac{h}{y}$.

• We again restrict attention to expenditure and tax rates that are constant over time

• $c(0) \rightarrow c(t) = c(0) \cdot e^{\gamma ht} \rightarrow h(t) = \tau_h \cdot y(0)e^{\gamma ht}$
There are then two first-order conditions corresponding to the maximization of $U$. Combining these leads to the familiar result: $\iota_g = g/y = \alpha$

That is, as long as $\iota_h = h/y$ is also chosen optimally, the optimal ratio for productive government expenditures is the same as before.
Self-interested Government

- Thus far, we assumed that the government was benevolent and therefore sought to maximize the utility attained by the representative household. I now consider the alternative that the government is run by an agent who has no electoral constraints and seeks to maximize his own utility.

\[ U = \int_0^\infty u(c)e^{-\rho t} dt \]

\[ u(c) = \frac{c^{1-\sigma}-1}{1-\sigma} \]
Self-interested Government

• the government agent has the same discount rate, $\rho$, as each household.

• The government agent uses his net revenue to purchase the quantity of consumer goods, $c_g$. The agent receives utility from consumption in the same manner as any household

\[
(26) \quad c_g = \left( \tau - \frac{g}{y} \right) \cdot y
\]

where the expenditure ratio $g/y$ can differ from the income tax rate $\tau$.
The government still uses a flat-rate income tax, but instead of automatically balancing the budget, the government can earn the net revenue.

With constant values for T and g/y, the privately determined growth rate is still the value \( y \) from equation (13).

\[
(13) \quad \gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} \cdot \left[ (1 - \tau) \cdot \phi \left( \frac{g}{k} \right) \cdot (1 - \eta) - \rho \right]
\]
Maximization Result

• For a Cobb-Douglas production function, the two first-order conditions for maximization of the agent's utility lead to the results:

\[ \tau > \frac{g}{y} = \alpha \, , \, \phi' = 1 \] (27)
Since the choice of $g/y$ is mainly one of efficient production, the self-interested government chooses the same value as the benevolent government. Basically, the government agent sets $g/y = \alpha$ to maximize the tax base that he has to work with. Then he is also in the position to set $\iota > g/y$ to secure the net flow of revenue, $c_g$. However, the ratio of government revenues to output exceeds $\alpha$ in both situations: in one case to provide consumption to the government agent and in the other to provide government consumption services to each household.
Some Empirical Implications
Governments randomized their actions.

Increase in the share of nonproductive governments expenditures

Differs $\alpha$s
Governments Randomized their actions

• The theory has implications for relations between the size of government and the rates of growth and saving.

• Because the analysis applies to steady-state growth paths, the natural empirical application would be to differences in average performance across countries over long periods of time.
Governments randomized their actions.

- As in usual in empirical investigations, the hypothesized effects of government policy are easier to assess if the government’s action can be treated as exogenous.
- That is, the result are simple if governments randomize their actions and thereby generate useful experimental data.
- As suggested before, productive government spending would include the resources devoted to property rights enforcement, as well as activities that enter directly into production functions.
Governments randomized their actions.

Countries could be arrayed along the horizontal axes by the size of $g/y$, and the responses of growth rate would be no monotonic, as shown in the figure...
Increase in the share of nonproductive governments expenditures

- An increase in the share of nonproductive government expenditures, leads to the types of shifts shown by the movements from the solid to dashed curves in figure ...
Increase in the share of nonproductive governments' expenditures

- For a given value of $g/y$, an increase in $h/y$, lowers the growth and saving rates.
- These effects arise because a higher $h/y$ has no direct effect on private-sector productivity, but does lead to a higher income tax rate.
- Since individuals retain a smaller fraction of their returns from investments, they have less incentive to invest, and the economy tends to grow at a lower rate.
Low incentives for investments

• The predictions are similar for any other differenced across countries that imply that private investors get to retain a smaller fraction of their returns from investments.

• If $g/y$, is held fixed, an increase in the average marginal tax rate or an exogenous worsening of property rights would tend to lower the growth and saving rates.
Asides from problems of measuring public services and the rates of growth and saving, the empirical implementation of the model is complicated by the endogeneity of the government.

Within the theatrical model, the governments sets the share of productive expenditures, $g/y$, to ensure productive efficiency.

Within this framework of optimizing governments, cross-sectional variations in $g/y$ arise only if $\alpha$ differs from country to country.
The Parameter \( \alpha \) which measures the productivity of public services relative to private services, could vary across countries for a number of reasons. These include geography, the share of agricultural production, urban density, and so on. The result is that a rise in \( \alpha \) and hence in \( g/y \), will reduce \( \gamma \).

The intuition is that an increase in \( \alpha \) means a shift in relative productivity toward the factor \( g \) that has to be financed by a distorting tax. It is for this reason that a higher \( \alpha \) correlates negatively with \( \gamma \).

The more general conclusion is that \( g/y \) and \( \gamma \) would show little correlation across countries because each government goes to point at which marginal effect of \( g/y \) on \( \gamma \) is close to zero.